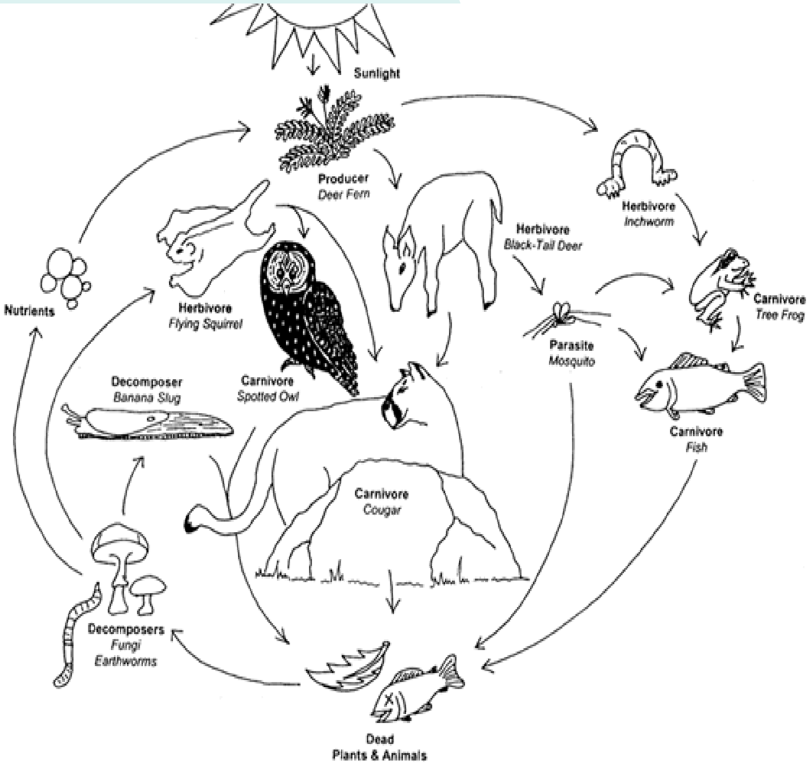
Simple Models of Complex Chaotic Systems



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Collaborators

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- **Konstantinos Chlouverakis**, *Univ Athens (Greece)*

Background

- Grew out of an multi-disciplinary chaos course that I taught 3 times
- Demands computation
- Strongly motivates students
- Used now for physics undergraduate research projects (~20 over the past 10 years)

Minimal Chaotic Systems

- 1-D map (quadratic map)

$$x_{n+1} = 1 - x_n^2$$

- Dissipative map (Hénon)

$$x_{n+1} = 1 - ax_n^2 + bx_{n-1}$$

- Autonomous ODE (jerk equation)

$$\dddot{x} + a\ddot{x} - \dot{x}^2 + x = 0$$

- Driven ODE (Ueda oscillator)

$$\ddot{x} + x^3 = \sin \omega t$$

- Delay differential equation (DDE)

$$\dot{x} = x_{t-\tau} - x_{t-\tau}^3$$

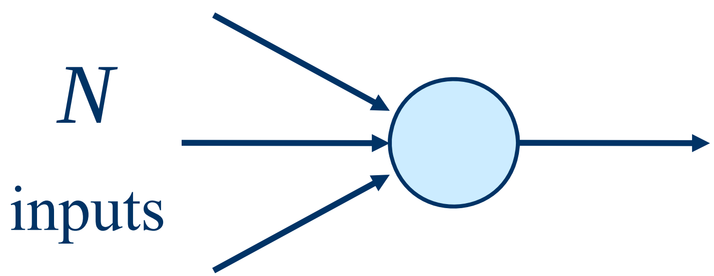
- Partial diff eqn (Kuramoto-Sivashinsky)

$$\partial_t u + u\partial_x u + a\partial_x^2 u + \partial_x^4 u = 0$$

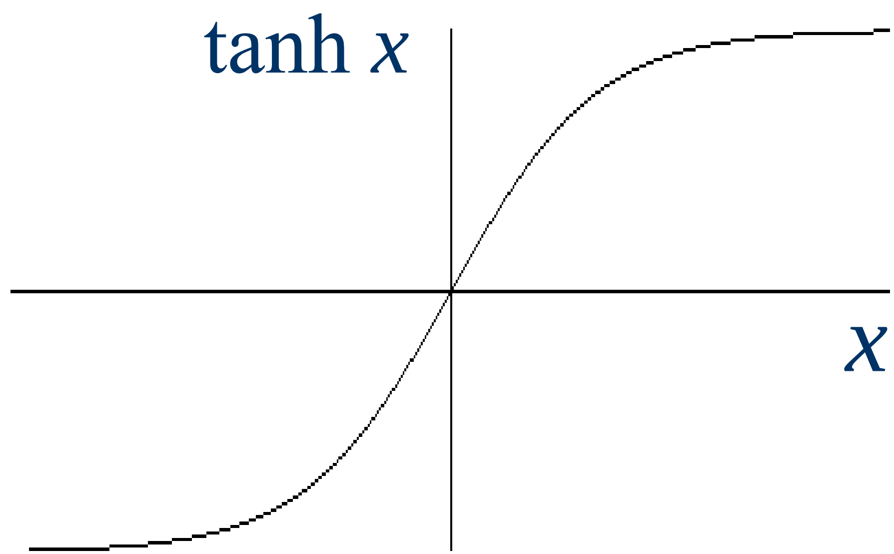
What is a complex system?

- Complex ≠ complicated
- Not real and imaginary parts
- Not very well defined
- Contains many interacting parts
- Interactions are nonlinear
- Contains feedback loops (+ and -)
- Cause and effect intermingled
- Driven out of equilibrium
- Evolves in time (not static)
- Usually chaotic (perhaps weakly)
- Can self-organize and adapt

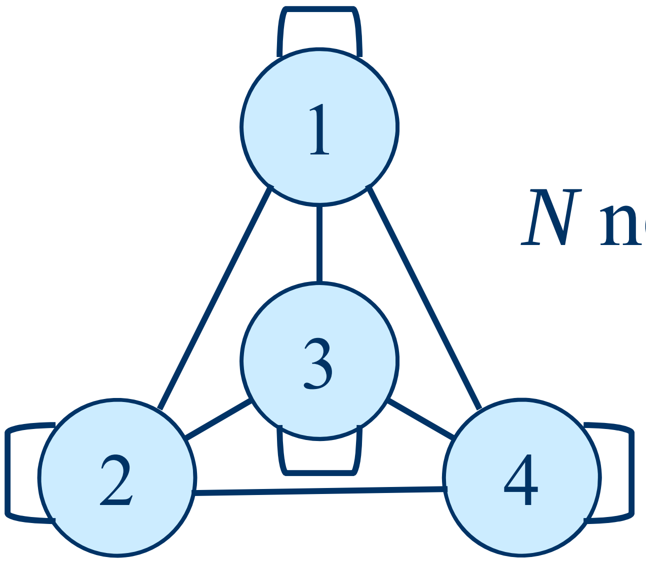
A Physicist's Neuron



$$x_{\text{out}} = \tanh \sum_{j=1}^{N} a_j x_j$$



A General Model (artificial neural network)

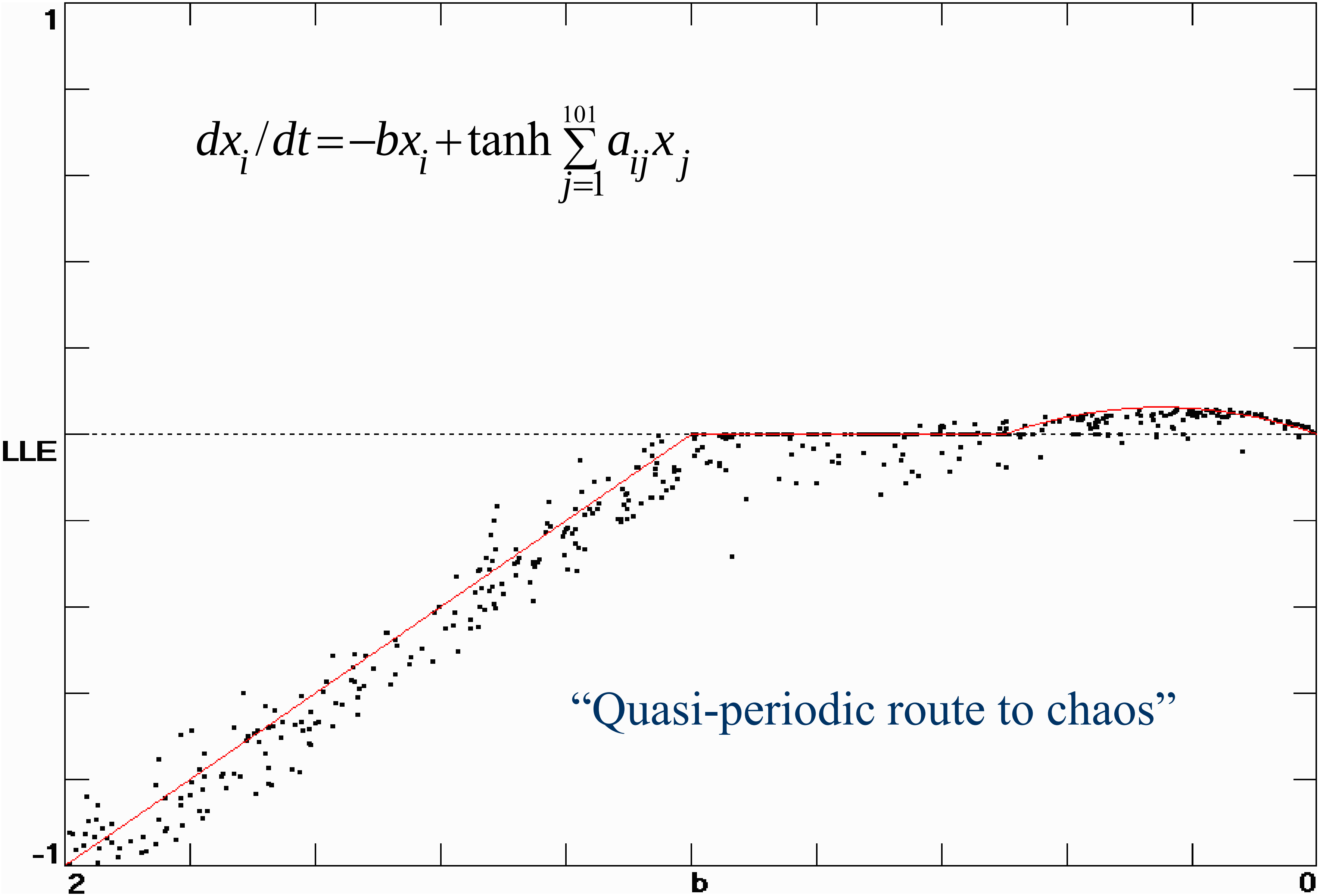


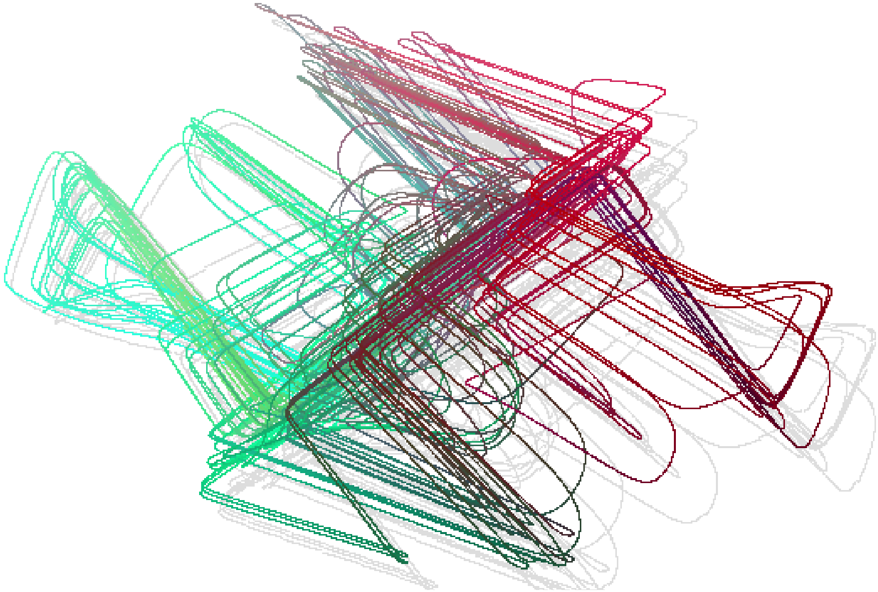
N neurons

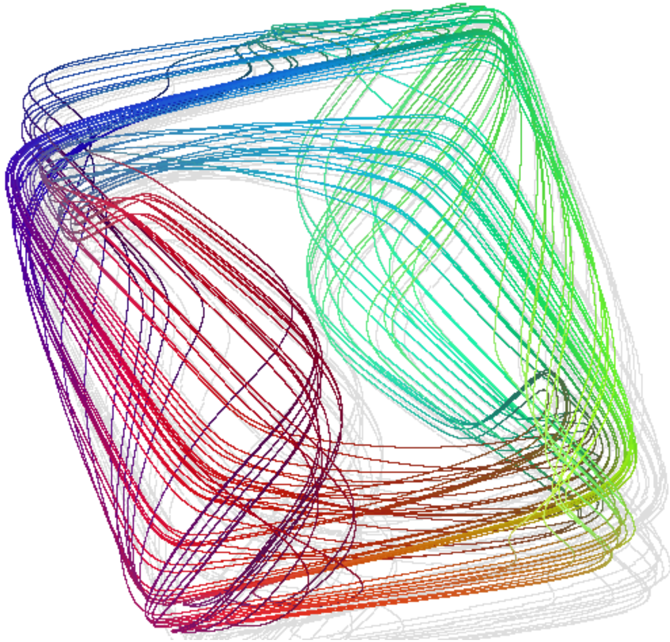
$$\dot{x}_i = -b_i x_i + \tanh \sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} x_j$$

"Universal approximator," $N \rightarrow \infty$

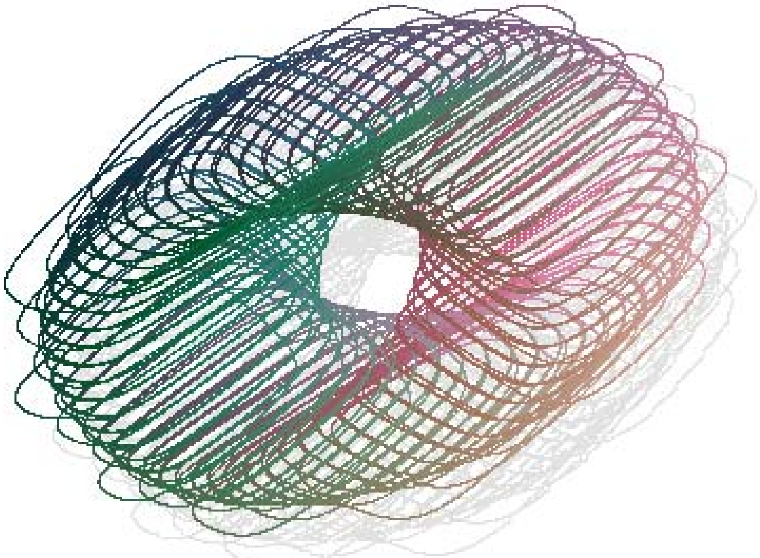
Route to Chaos at Large *N* (=101)

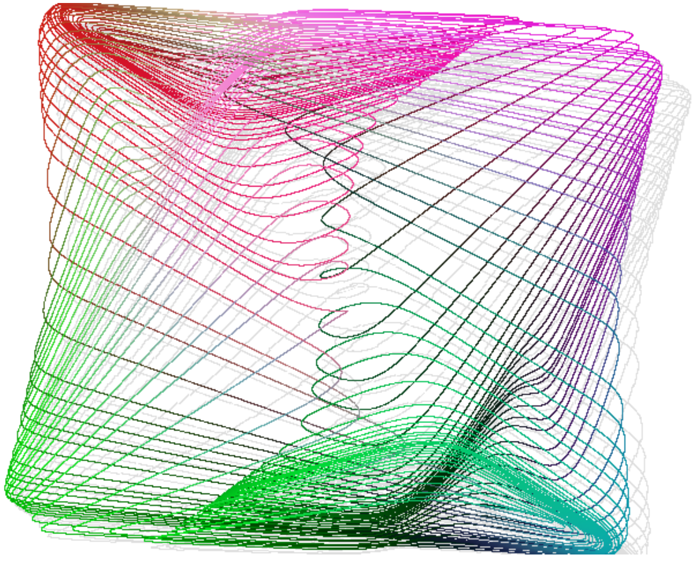




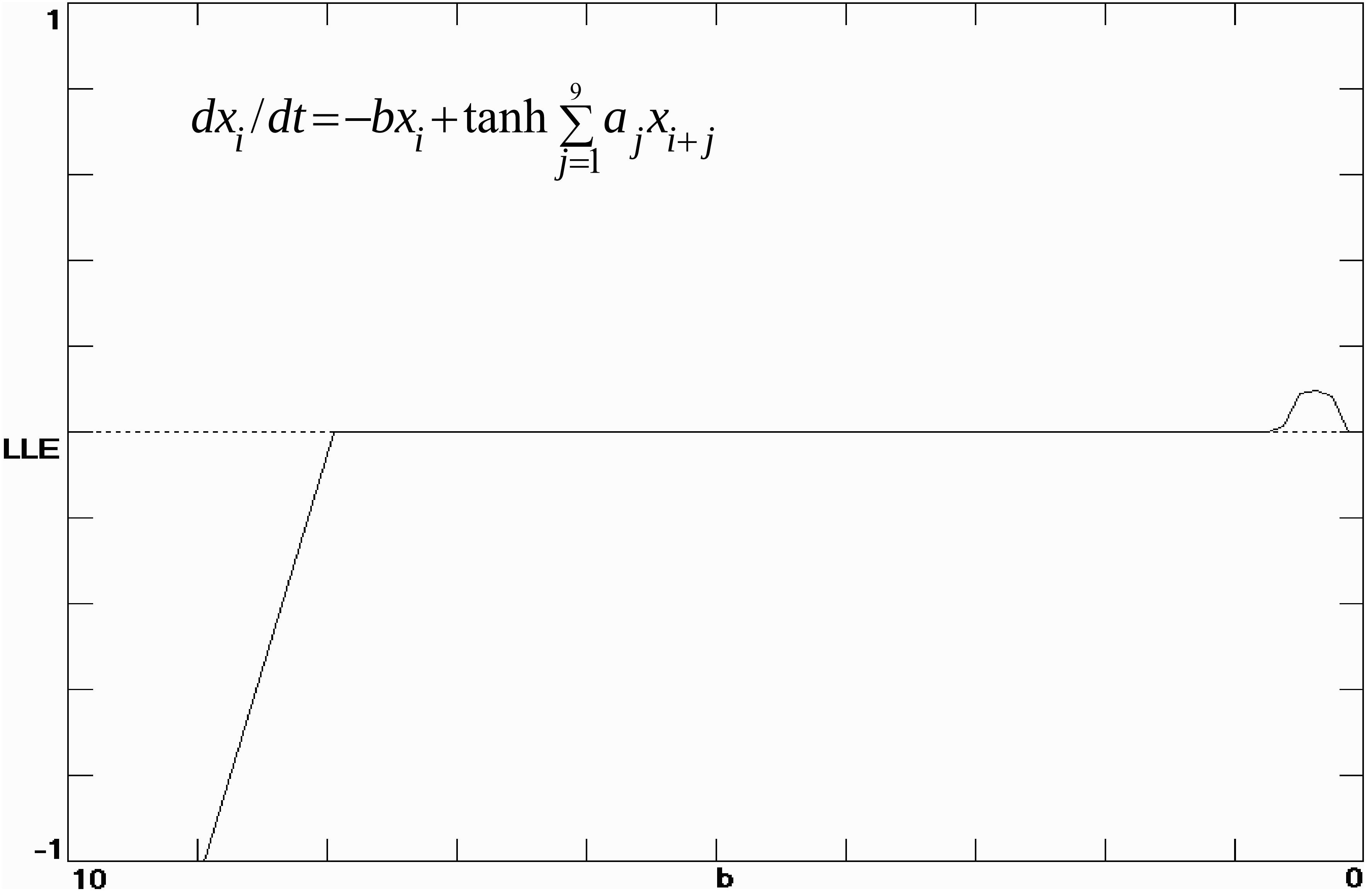


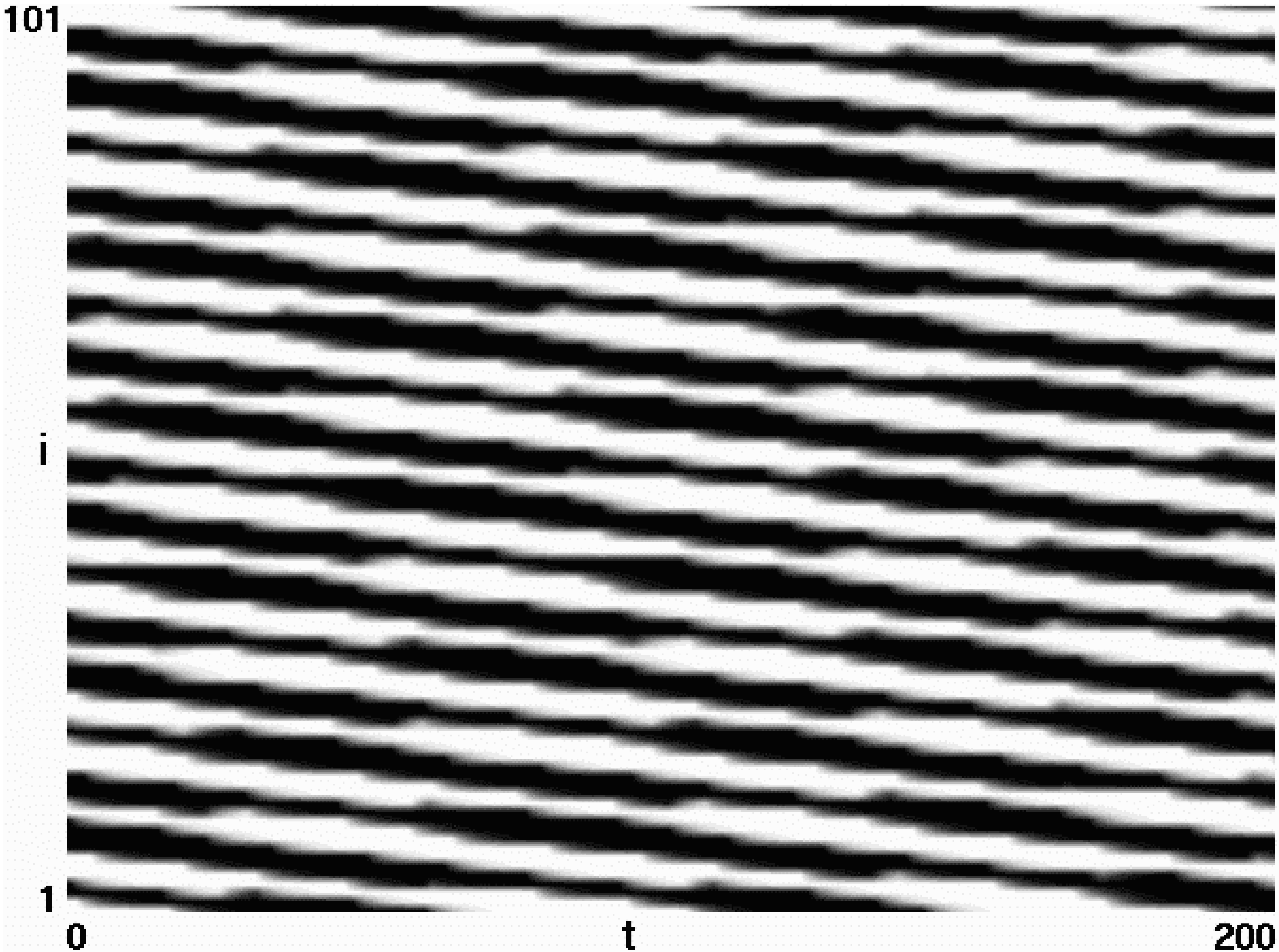
Strange Attractors



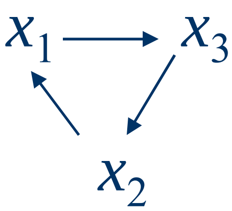


Sparse Circulant Network (N=101)




101
i
1
0
t
200

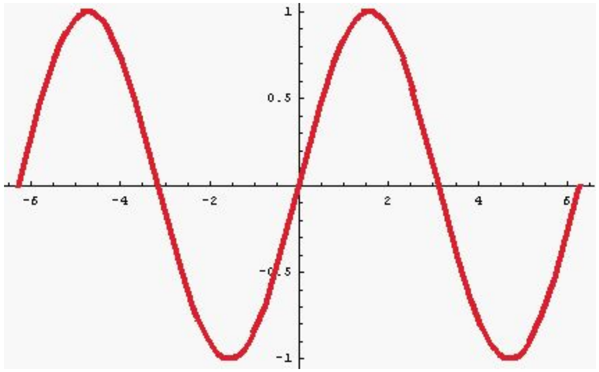
Labyrinth Chaos

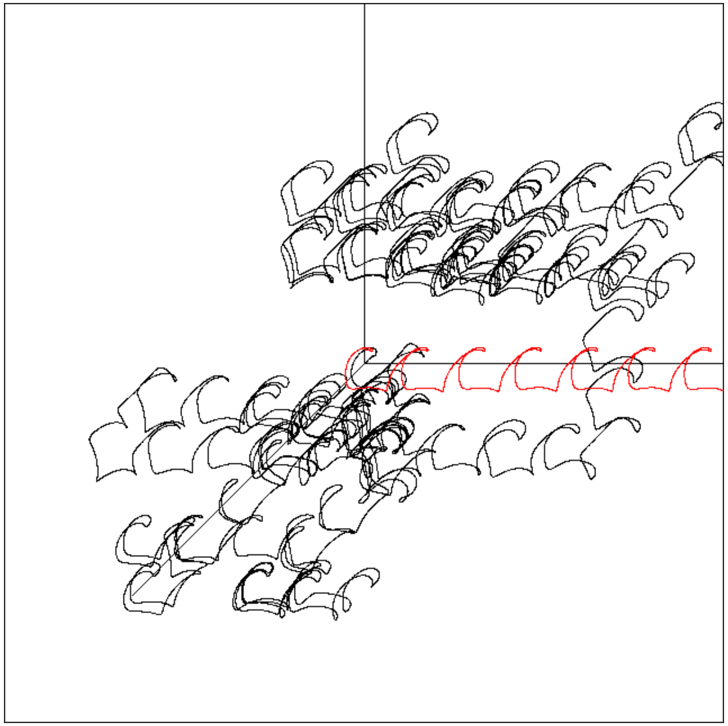


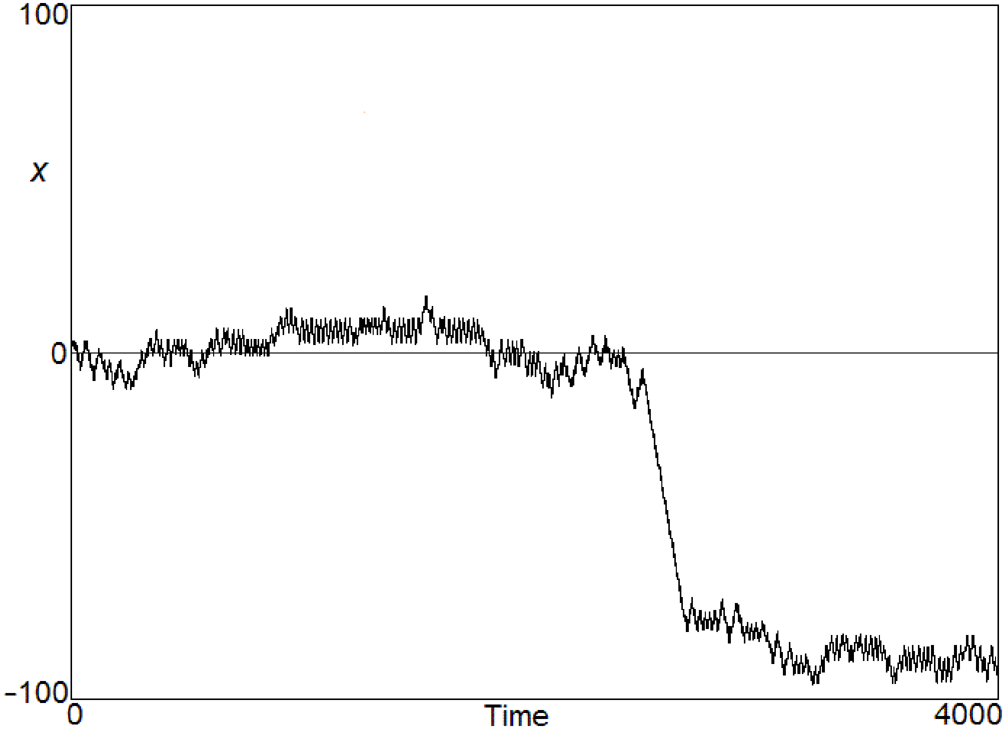
$$dx_1/dt = \sin x_2$$

$$dx_2/dt = \sin x_3$$

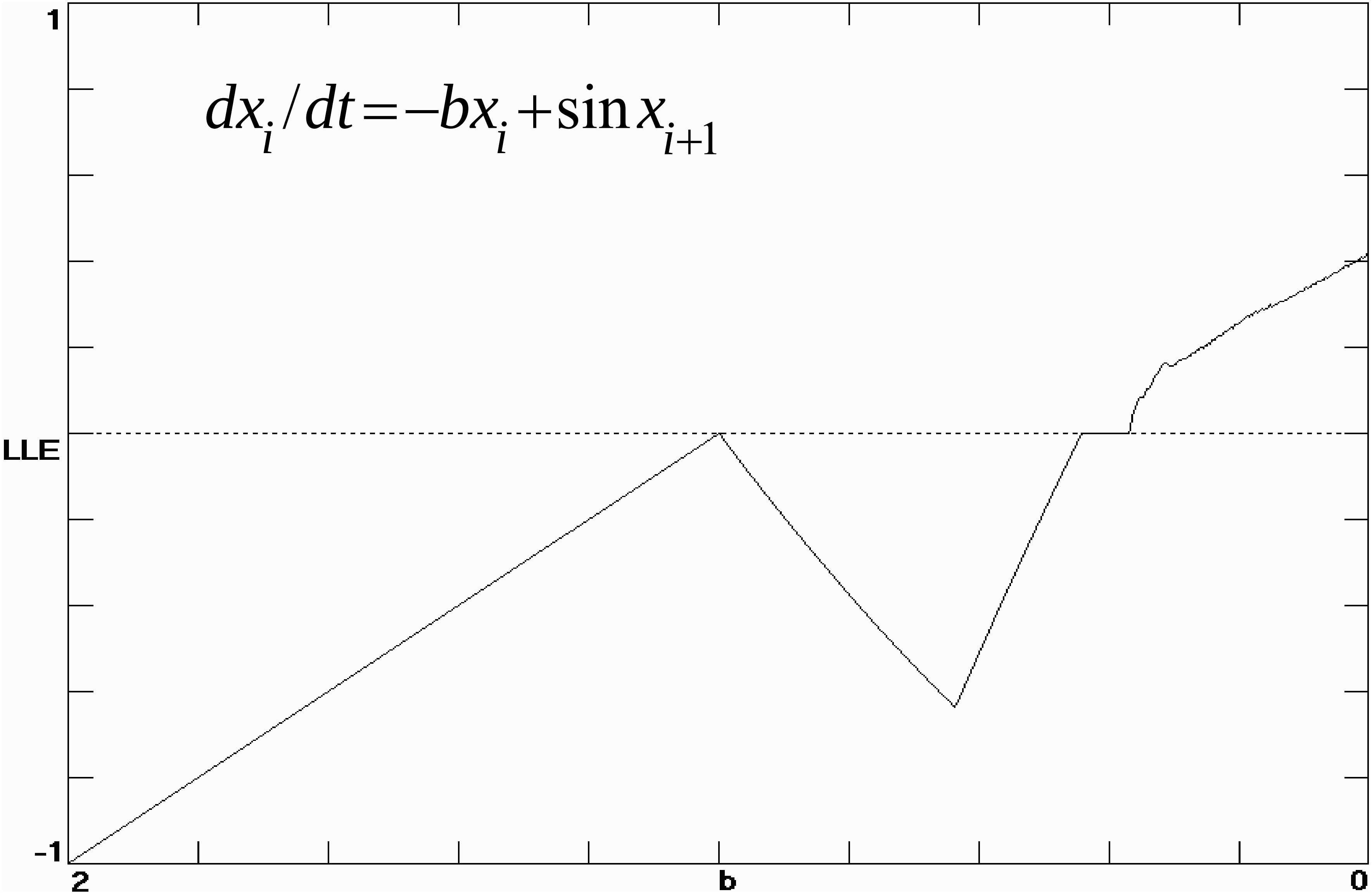
$$dx_3/dt = \sin x_1$$

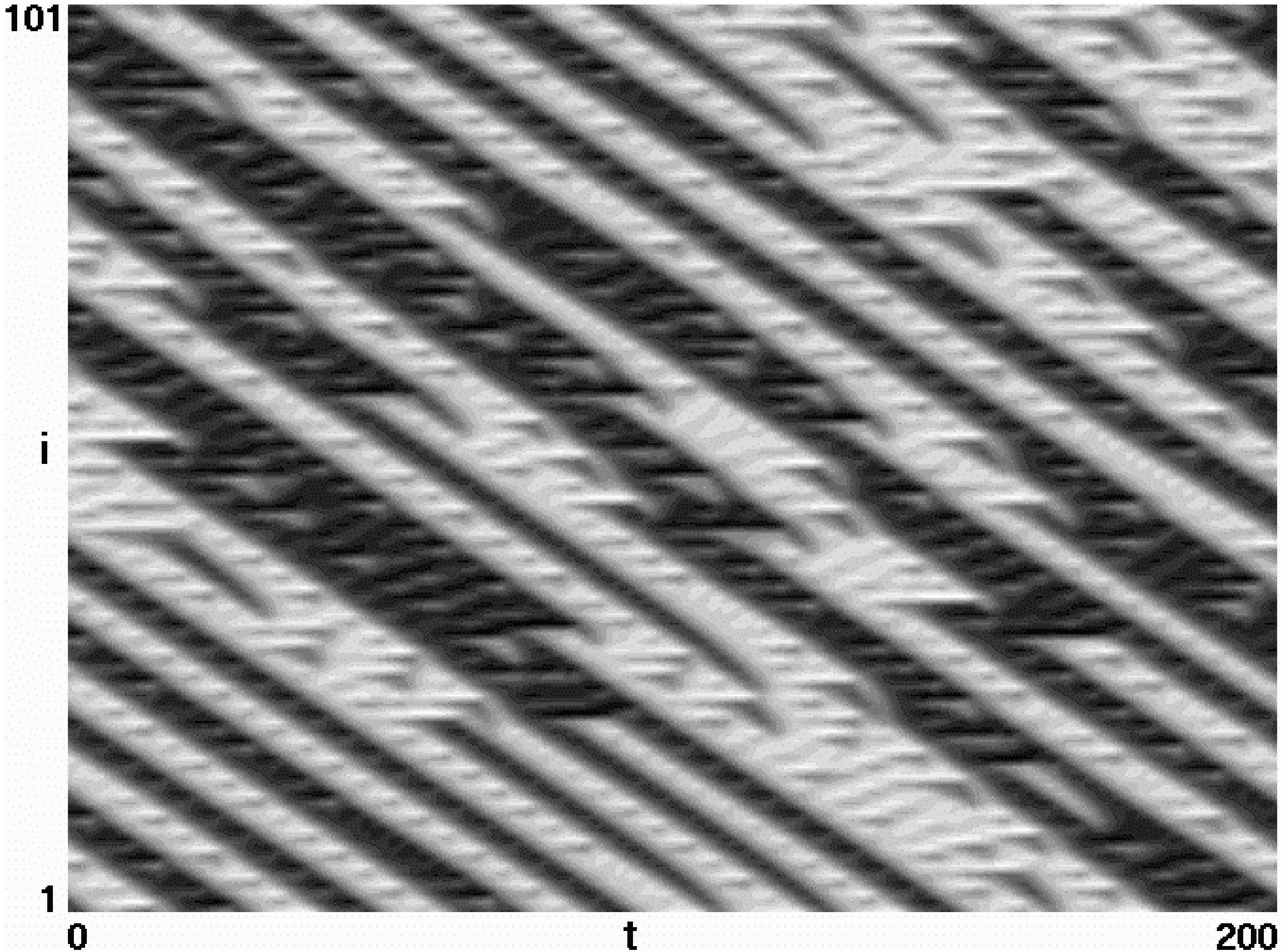






Hyperlabyrinth Chaos (N=101)

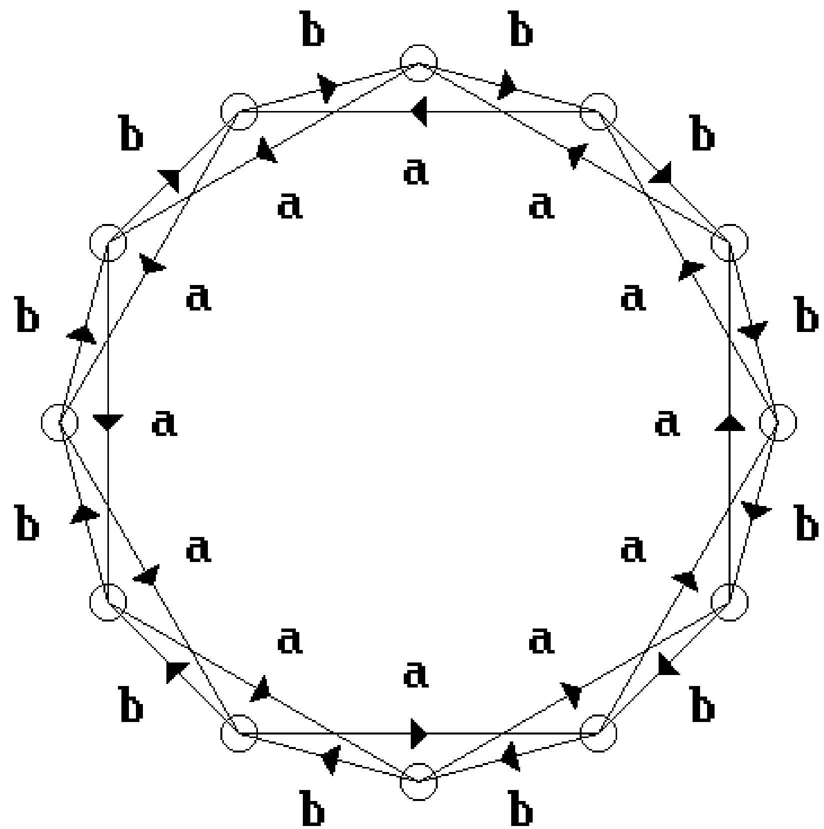




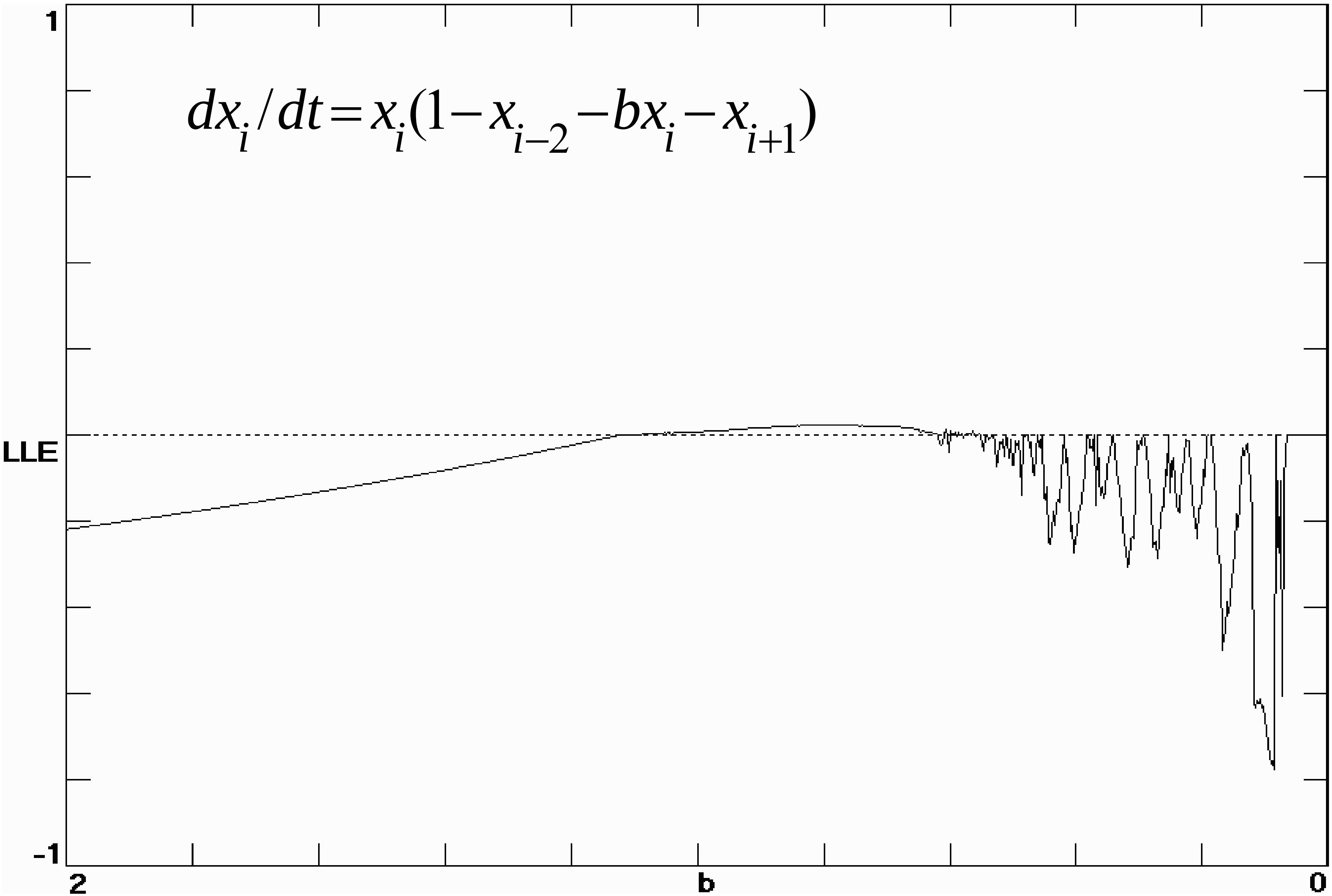
101
i
1
0
t
200

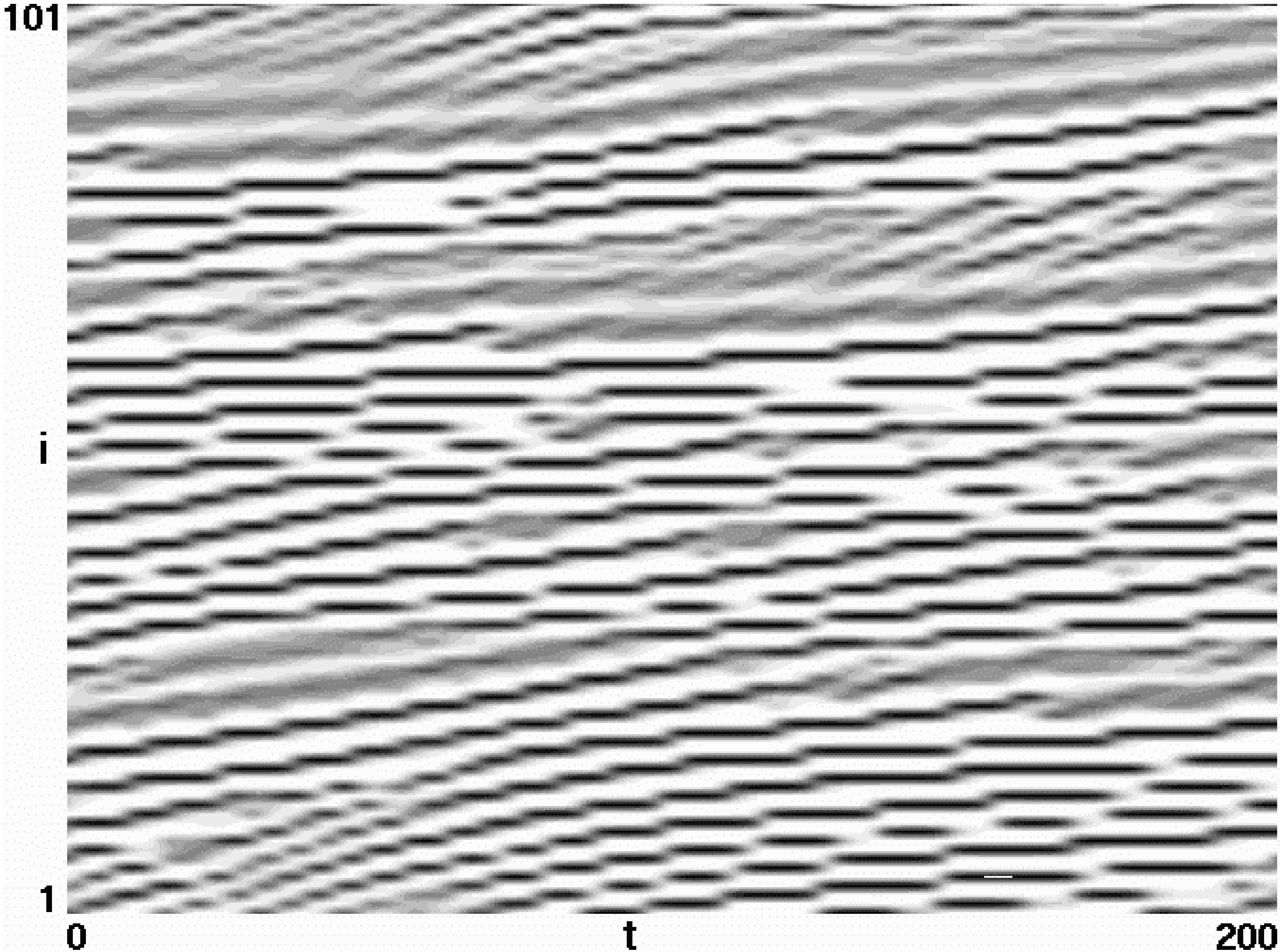
Minimal High-D Chaotic L-V Model

$$dx_i/dt = x_i(1 - x_{i-2} - x_i - x_{i+1})$$



Lotka-Volterra Model (N=101)

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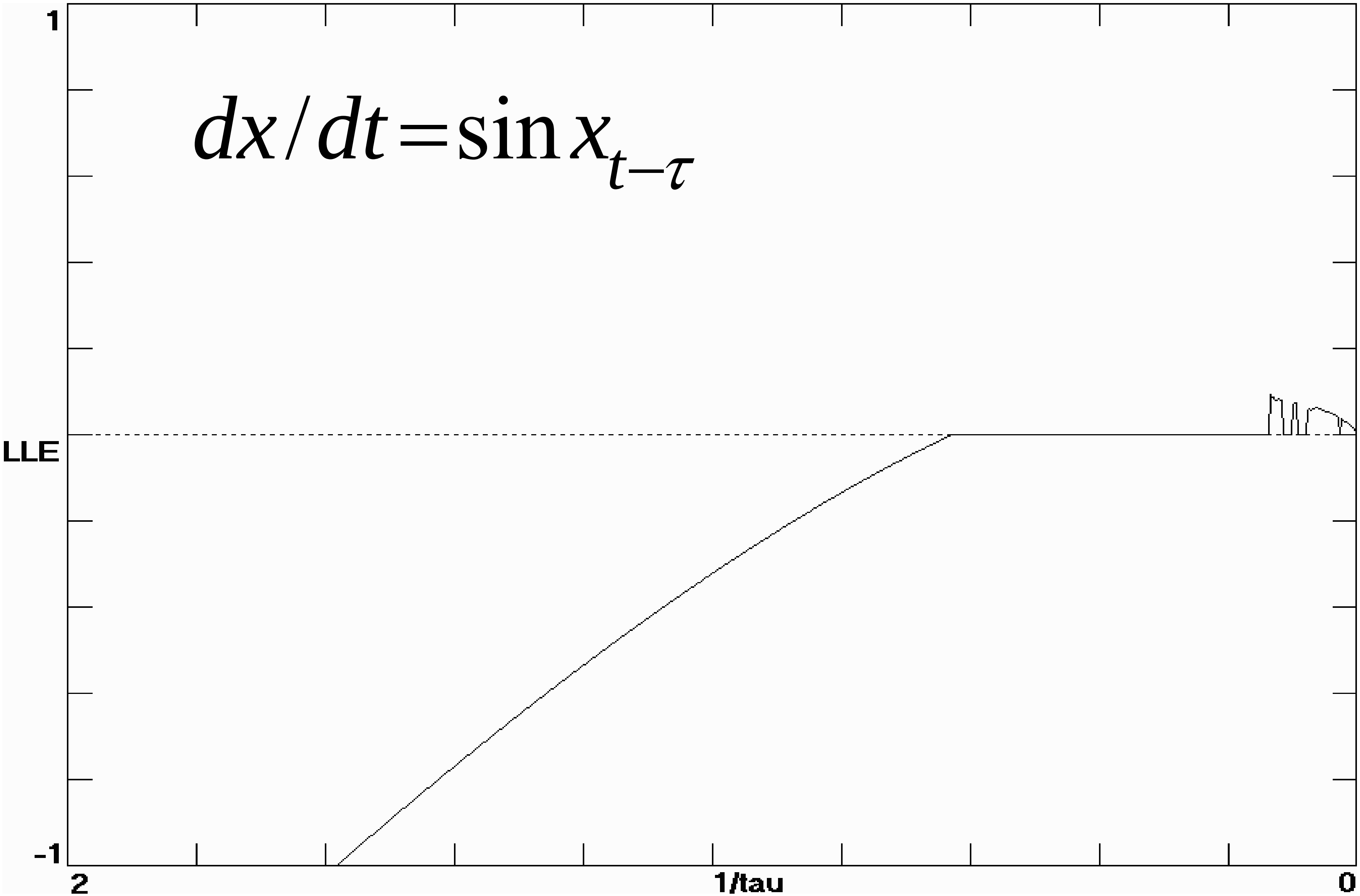
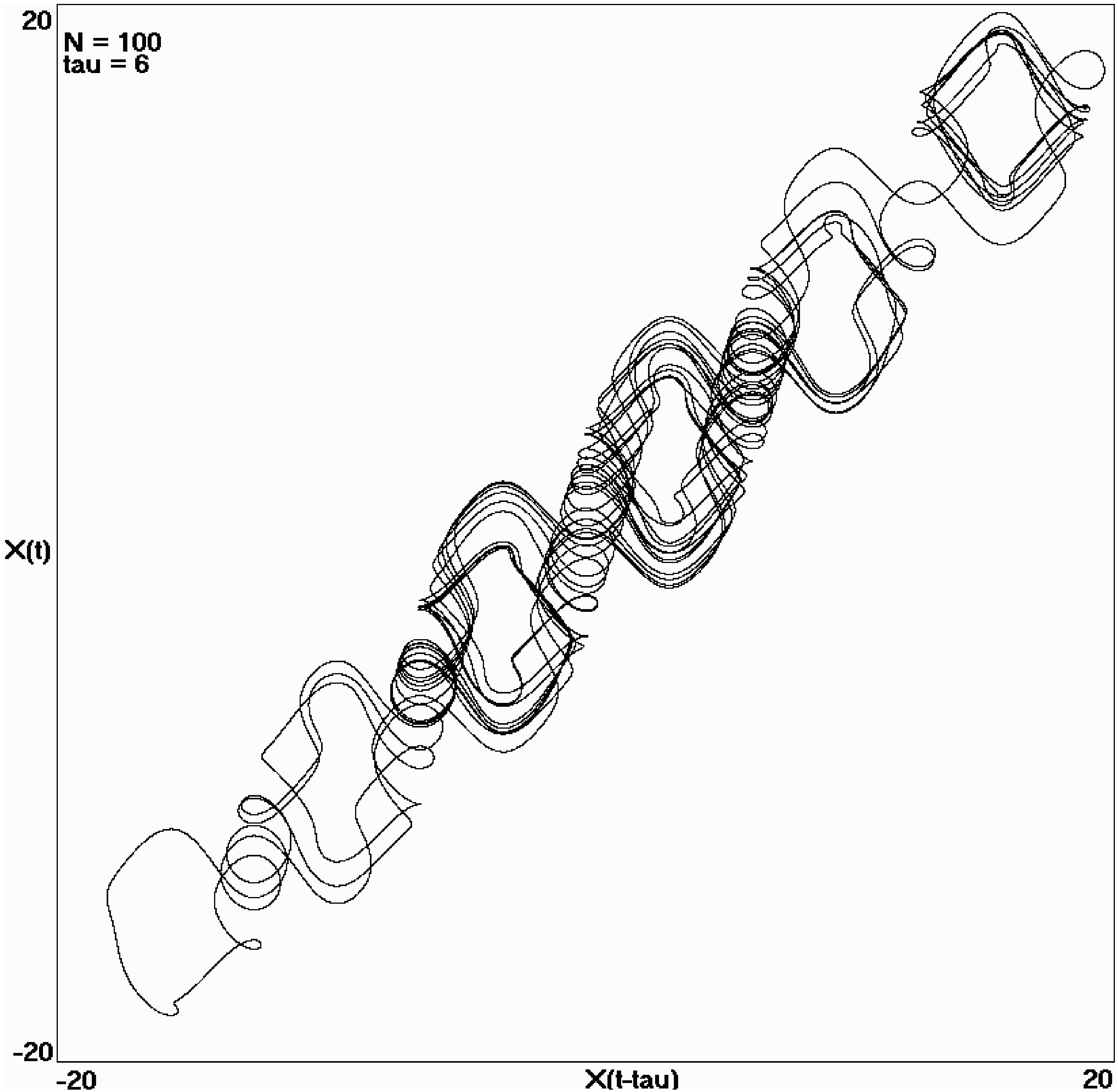
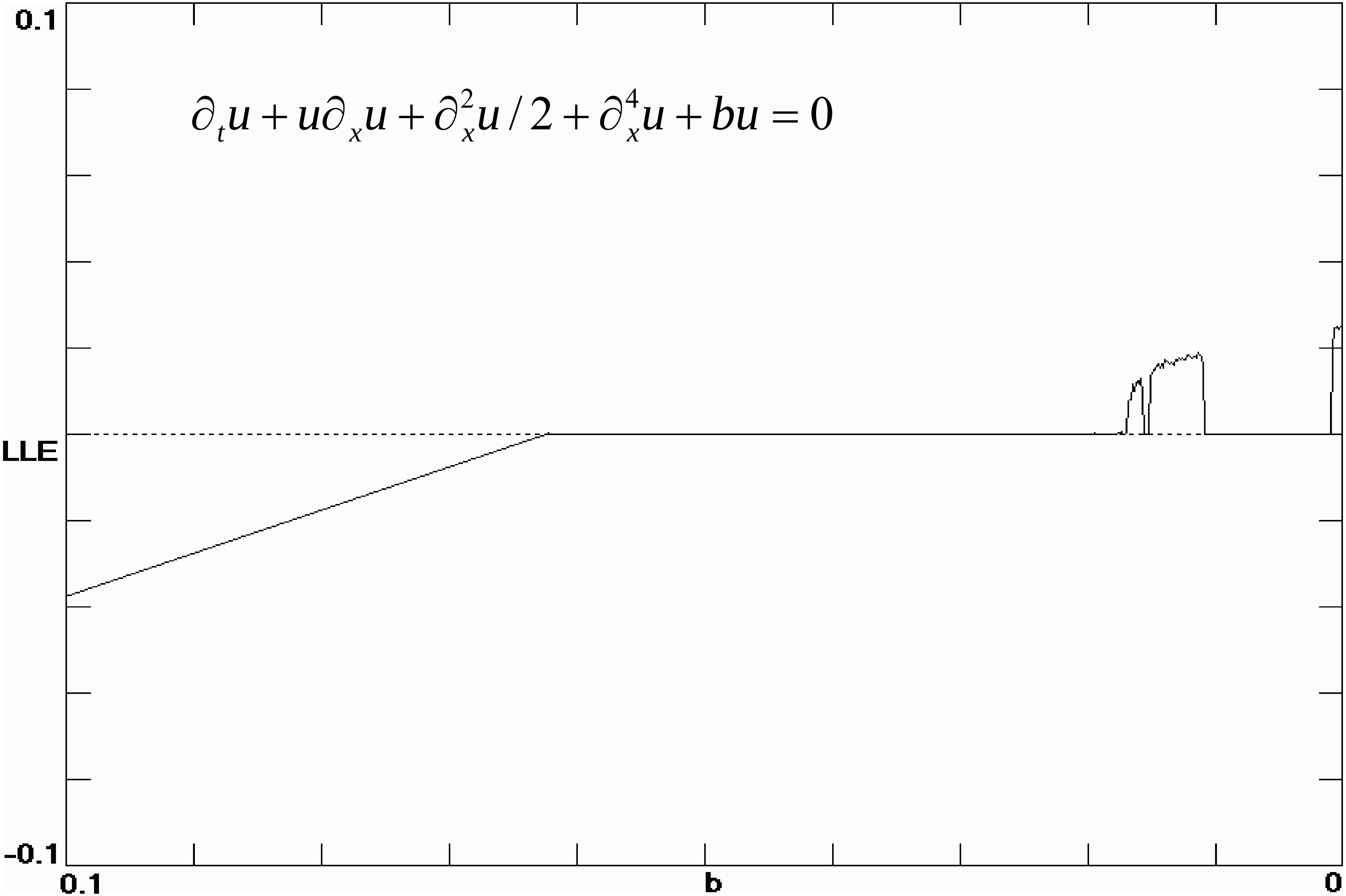
Delay Differential Equation

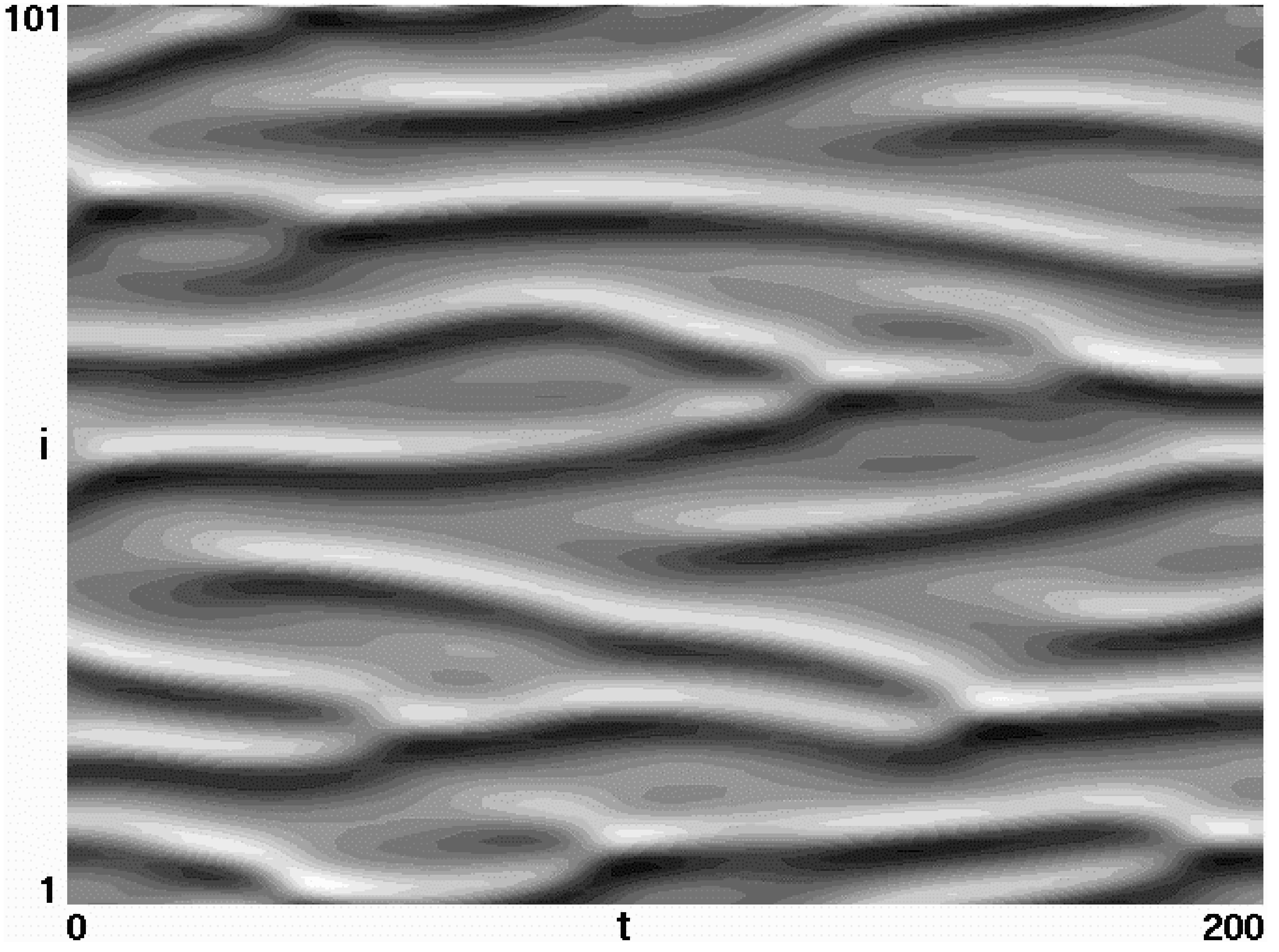




20
N = 100
tau = 6
X(t)
-20
-20
X(t-tau)
20

Partial Differential Equation



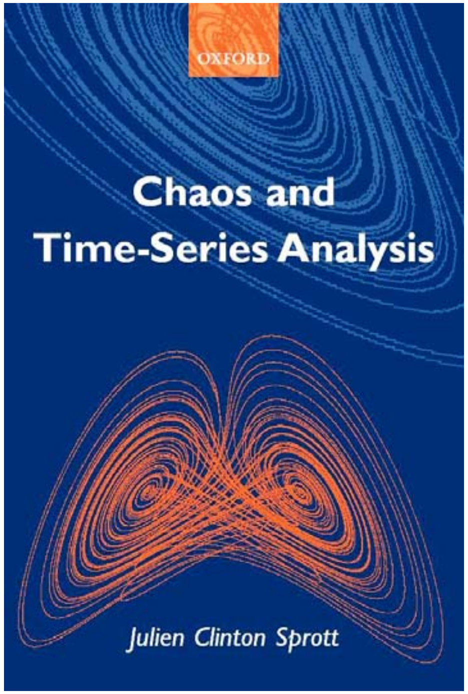

101
i
1
0
t
200

Summary of High-N Dynamics

- Chaos is common for highly-connected networks
- Sparse, circulant networks can also be chaotic (but the parameters must be carefully tuned)
- Quasiperiodic route to chaos is usual
- Symmetry-breaking, self-organization, pattern formation, and spatio-temporal chaos occur
- Maximum attractor dimension is of order $N/2$
- Attractor is sensitive to parameter perturbations, but dynamics are not

Shameless Plug

Chaos and Time-Series Analysis
J. C. Sprott
Oxford University Press (2003)
ISBN 0-19-850839-5



An introductory text for advanced undergraduate and beginning graduate students in all fields of science and engineering

References

- http://sprott.physics.wisc.edu/lectures/davidson.ppt (this talk)
- http://sprott.physics.wisc.edu/chaostsa/ (my chaos textbook)
- sprott@physics.wisc.edu (contact me)